Exercise 6.1 (Revised) - Chapter 6 - Triangles - Ncert Solutions class 10 - Maths

Updated On 11-02-2025 By Lithanya

Chapter 6 Triangles NCERT Solutions Class 10 Maths: Simplify Concepts & Ace Exams

Ex 6.1 Question 1.

Fill in the blanks using the correct word given in brackets:
(i) All circles are ______________. (congruent, similar)
(ii) All squares are ___________________. (similar, congruent)
(iii) All ______________ triangles are similar. (isosceles, equilateral)
(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are ______________ and (b) their corresponding sides are _____________ . (equal, proportional)

Answer.

(i) similar
(ii) similar
(iii) equilateral
(iv) equal, proportional

Ex 6.1 Question 2.

Give two different examples of pair of:
(i) similar figures
(ii) non-similar figures

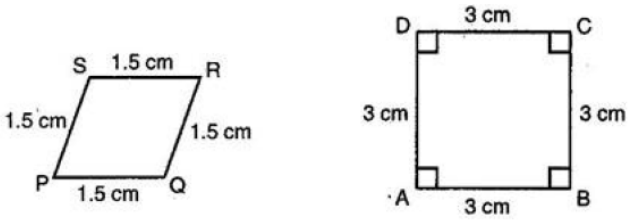
Answer.

(i) Two different examples of a pair of similar figures are:
(a) Any two rectangles
(b) Any two squares
(ii) Two different examples of a pair of non-similar figures are:

(a) A scalene and an equilateral triangle
(b) An equilateral triangle and a right angled triangle

Ex 6.1 Question 3.

State whether the following quadrilaterals are similar or not:



Answer.

On looking at the given figures of the quadrilaterals, we can say that they are not similar because their angles are not equal.

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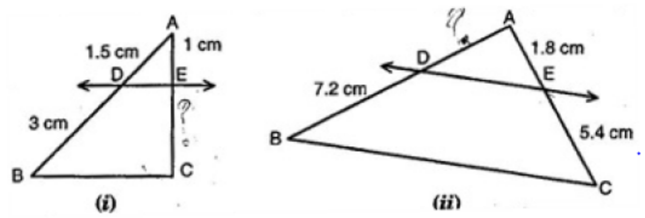
Ex 6.2 Question 1.

In figure (i) and (ii), DE||BC. Find EC in (i) and AD in (ii).



Answer.

(i) Since $DE \| BC$,

$\therefore \frac{\mathrm{AD}}{\mathrm{DB}}=\frac{\mathrm{AE}}{\mathrm{EC}}$

$\Rightarrow \frac{1.5}{3}=\frac{1}{\mathrm{EC}}$

$\Rightarrow \mathrm{EC}=\frac{3}{1.5}$

$\Rightarrow \mathrm{EC}=2 \mathrm{~cm}$

(ii)Since $DE \| BC$,

$\therefore \frac{\mathrm{AD}}{\mathrm{DB}}=\frac{\mathrm{AE}}{\mathrm{EC}}$

$\Rightarrow \frac{\mathrm{AD}}{7.2}=\frac{1.8}{5.4}$

$\Rightarrow \mathrm{AD}=\frac{1.8 \times 7.2}{5.4}$

$\Rightarrow \mathrm{EC}=2.4 \mathrm{~cm}$

Ex 6.2 Question 2.

E and F are points on the sides PQ and PR respectively of a $\triangle PQR$. For each of the following cases, state whether $EF \| Q$ Q:

(i) $\mathrm{PE}=3.9 \mathrm{~cm}, \mathrm{EQ}=4 \mathrm{~cm}, \mathrm{PF}=3.6 \mathrm{~cm}$ and $\mathrm{FR}=2.4 \mathrm{~cm}$

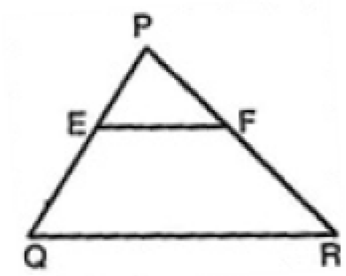
(ii) $\mathrm{PE}=4 \mathrm{~cm}, \mathrm{QE}=4.5 \mathrm{~cm}, \mathrm{PF}=8 \mathrm{~cm}$ and $\mathrm{RF}=9 \mathrm{~cm}$

(iii) $\mathrm{PQ}=1.28 \mathrm{~cm}, \mathrm{PR}=2.56 \mathrm{~cm}, \mathrm{PE}=0.18 \mathrm{~cm}$ and $\mathrm{PF}=0.36 \mathrm{~cm}$

Answer.

(i)Given: $\mathrm{PE}=3.9 \mathrm{~cm}, \mathrm{EQ}=4 \mathrm{~cm}, \mathrm{PF}=3.6 \mathrm{~cm}$ and $\mathrm{FR}=2.4 \mathrm{~cm}$

Now, $\frac{\mathrm{PE}}{\mathrm{EQ}}=\frac{3.9}{4}=0.97 \mathrm{~cm}$



And $\frac{PF}{FR} = \frac{3.6}{2.4} = 1.2 \text{ cm}$

$\because \frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF does not divide the sides PQ and PR of $\triangle PQR$ in the same ratio.

$\therefore$ EF is not parallel to QR.

(ii) Given: $PE = 4 \text{ cm}, QE = 4.5 \text{ cm}, PF = 8 \text{ cm}$ and $RF = 9 \text{ cm}$

Now, $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9} \text{cm}$

And $\frac{PF}{FR} = \frac{8}{9} \text{cm}$

$\because \frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF divides the sides PQ and PR of $\triangle PQR$ in the same ratio.

$\therefore$ EF is parallel to QR.

(iii) Given: $PQ = 1.28 \text{ cm}, PR = 2.56 \text{ cm}, PE = 0.18 \text{ cm}$ and $PF = 0.36 \text{ cm}$

$\Rightarrow EQ = PQ - PE = 1.28 - 0.18 = 1.10 \text{ cm}$

And $ER = PR - PF = 2.56 - 0.36 = 2.20 \text{ cm}$

Now, $\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55} \text{cm}$

And $\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55} \text{cm}$

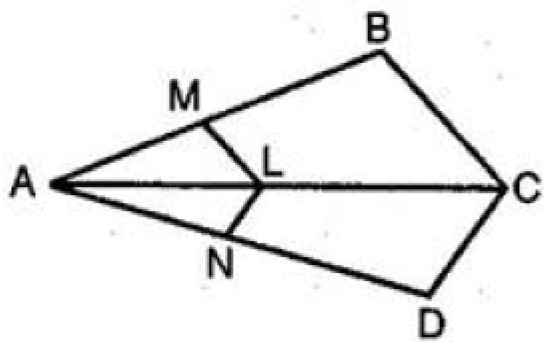
$\because \frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF divides the sides PQ and PR of $\triangle PQR$ in the same ratio.

$\therefore$ EF is parallel to QR.

Ex 6.2 Question 3.

In figure, if $LM \| CB$ and $LN \| CD$, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Answer.

In $\triangle ABC, LM \| CB$

$\therefore \frac{AM}{AB} = \frac{AL}{AC}$ [Basic Proportionality theorem]

And in $\triangle ACD, LN \| CD$

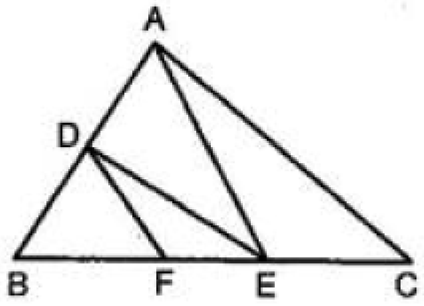
$\therefore \frac{AL}{AC} = \frac{AN}{AD}$ [Basic Proportionality theorem]

From eq. (i) and (ii), we have

$\frac{AM}{AB} = \frac{AN}{AD}$

Ex 6.2 Question 4.

In the given figure, $DE \| AC$ and $DF \| AE$. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



Answer.

In $\triangle BCA, DE \| AC$

$\therefore \frac{BE}{EC} = \frac{BD}{DA}$ [Basic Proportionality theorem]

And in $\triangle BEA, DF \| AE$

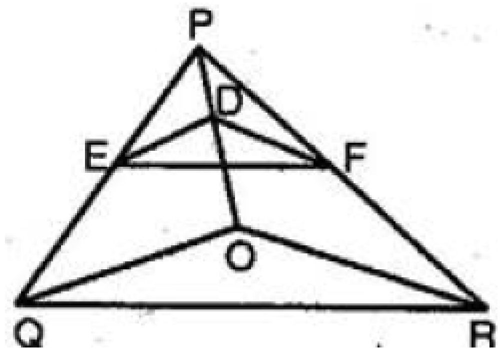
$\therefore \frac{BE}{FE} = \frac{BD}{DA}$[Basic Proportionality theorem]

From eq. (i) and (ii), we have

$\frac{\mathrm{BF}}{\mathrm{FE}}=\frac{\mathrm{BE}}{\mathrm{EC}}$

Ex 6.2 Question 5.

In the given figure, DE $\| OQ$ and DF $\|$ OR. Show that EF $\| QR$.



Answer.

In $\triangle \mathrm{PQO}, \mathrm{DE} \| \mathrm{OQ}$

$\therefore \frac{\mathrm{PE}}{\mathrm{EQ}}=\frac{\mathrm{PD}}{\mathrm{DO}}$[Basic Proportionality theorem]

And in $\triangle \mathrm{POR}, \mathrm{DF} \| \mathrm{OR}$

$\therefore \frac{\mathrm{PD}}{\mathrm{DO}}=\frac{\mathrm{PF}}{\mathrm{FR}}$[Basic Proportionality theorem]

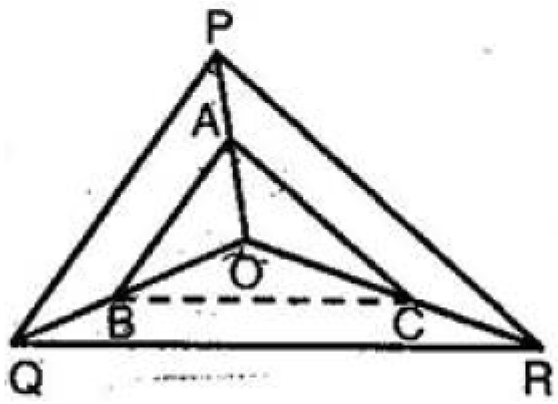
From eq. (i) and (ii), we have

$\frac{\mathrm{PE}}{\mathrm{EQ}}=\frac{\mathrm{PF}}{\mathrm{FR}}$

$\therefore \mathrm{EF} \| \mathrm{QR}$[By the converse of BPT]

Ex 6.2 Question 6.

In the given figure, A, B, and C are points on OP, OQ and OR respectively such that $AB \| PQ$ and $AC \| PR$. Show that $BC \|$ QR.



Answer.

Given: O is any point in $\triangle \mathrm{PQR}$, in which $\mathrm{AB} \| \mathrm{PQ}$ and $\mathrm{AC} \| \mathrm{PR}$.

To prove: $BC \| QR$

Construction: Join BC.

Proof: In $\triangle \mathrm{OPQ}, \mathrm{AB} \| \mathrm{PQ}$

$\therefore \frac{OA}{AP}=\frac{OB}{BQ}$[Basic Proportionality theorem]

And in $\triangle \mathrm{OPR}, \mathrm{AC} \| \mathrm{PR}$

$\therefore \frac{\mathrm{OA}}{\mathrm{AP}}=\frac{\mathrm{OC}}{\mathrm{CR}}$[Basic Proportionality theorem].

From eq. (i) and (ii), we have

$\frac{OB}{BQ}=\frac{OC}{CR}$

$\because$ In $\triangle \mathrm{OQR}, \mathrm{B}$ and C are points dividing the sides OQ and OR in the same ratio.

$\therefore$ By the converse of Basic Proportionality theorem,

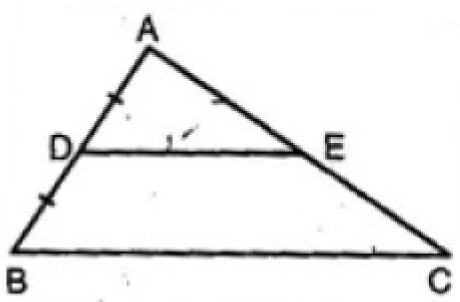
$\Rightarrow \mathrm{BC} \| \mathrm{QR}$

Ex 6.2 Question 7.

Using Theorem 6.1, prove that a line drawn through the midpoint of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer.

Given: A triangle ABC, in which D is the midpoint of side AB and the line DE is drawn parallel to BC, meeting AC at E.



To prove: $\mathrm{AE}=\mathrm{EC}$
Proof: Since DE $\|$ BC
$\therefore \frac{\mathrm{AD}}{\mathrm{DB}}=\frac{\mathrm{AE}}{\mathrm{EC}}$ [Basic Proportionality theorem]
But $\mathrm{AD}=\mathrm{DB}[$ Given $]$

$$\Rightarrow \frac{\mathrm{AD}}{\mathrm{DB}}=1$$

$$\Rightarrow \frac{\mathrm{AE}}{\mathrm{EC}}=1 \text{ [From eq. (i)]}$$

$$\Rightarrow \mathrm{AE}=\mathrm{EC}$$

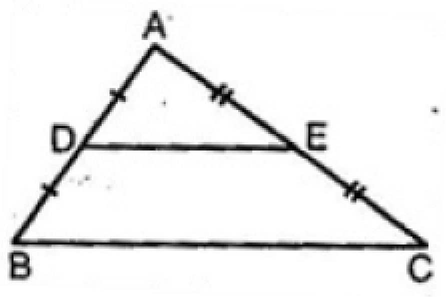
Hence, E is the midpoint of the third side AC.

Ex 6.2 Question 8.

Using Theorem 6.2, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer.

Given: A triangle ABC, in which D and E are the midpoints of sides AB and AC respectively.



To Prove: DE \| BC
Proof: Since D and E are the midpoints of AB and AC
respectively.
$\therefore \mathrm{AD}=\mathrm{DB}$ and $\mathrm{AE}=\mathrm{EC}$
Now, $\mathrm{AD}=\mathrm{DB}$

$$\Rightarrow \frac{\mathrm{AD}}{\mathrm{DB}}=1 \text{ and } \mathrm{AE}=\mathrm{EC}$$

$$\Rightarrow \frac{\mathrm{AE}}{\mathrm{EC}}=1$$

$$\therefore \frac{\mathrm{AD}}{\mathrm{DB}}=\frac{\mathrm{AE}}{\mathrm{EC}}=1$$

$$\Rightarrow \frac{\mathrm{AD}}{\mathrm{DB}}=\frac{\mathrm{AE}}{\mathrm{EC}}$$

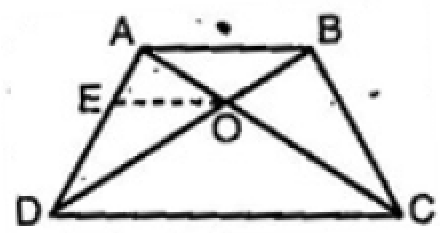
Thus, in triangle ABC, D and E are points dividing the sides AB and AC in the same ratio.
Therefore, by the converse of Basic Proportionality theorem, we have
$\mathrm{DE} \| \mathrm{BC}$

Ex 6.2 Question 9.

$ABCD$ is a trapezium in which $AB \| DC$ and its diagonals intersect each other at the point 0 . Show that $\frac{\mathrm{AO}}{\mathrm{BO}}=\frac{\mathrm{CO}}{\mathrm{DO}}$.

Answer.

Given: A trapezium ABCD, in which $\mathrm{AB} \| \mathrm{DC}$ and its diagonals
AC and BD intersect each other at O.



To Prove: $\frac{AO}{BO}=\frac{CO}{DO}$
Construction: Through O, draw OE $\| \mathrm{AB}$, i.e. OE $\|$ DC.
Proof: In $\triangle \mathrm{ADC}$, we have $\mathrm{OE} \| \mathrm{DC}$
$\therefore \frac{\mathrm{AE}}{\mathrm{ED}}=\frac{\mathrm{AO}}{\mathrm{CO}}$ [By Basic Proportionality theorem]
Again, in $\triangle \mathrm{ABD}$, we have $\mathrm{OE} \| \mathrm{AB}[$ Construction $]$
$\therefore \frac{\mathrm{ED}}{\mathrm{AE}}=\frac{\mathrm{DO}}{\mathrm{BO}}$ [By Basic Proportionality theorem]
$\Rightarrow \frac{AE}{ED}=\frac{BO}{DO}$

From eq. (i) and (ii), we get

$$\frac{AO}{CO}=\frac{BO}{DO}$$

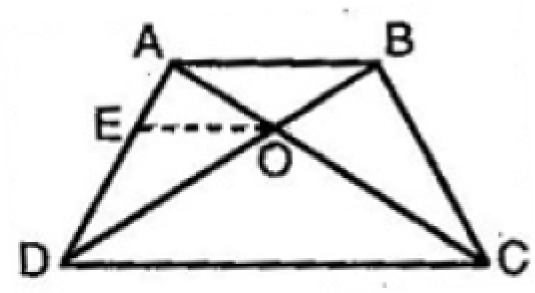
$$\Rightarrow \frac{AO}{BO}=\frac{CO}{DO}$$

Ex 6.2 Question 10.

The diagonals of a quadrilateral $ABCD$ intersect each other at the point O such that $\frac{AO}{BO}=\frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer.

Given: A quadrilateral ABCD, in which its diagonals AC and BD intersect each other at O such that $\frac{AO}{BO}=\frac{CO}{DO}$, i.e.



$\frac{\mathrm{AO}}{\mathrm{CO}}=\frac{\mathrm{BO}}{\mathrm{DO}}$

To Prove: Quadrilateral ABCD is a trapezium.

Construction: Through O, draw $\mathrm{OE} \| \mathrm{AB}$ meeting AD at E.

Proof: In $\triangle \mathrm{ADB}$, we have $\mathrm{OE} \| \mathrm{AB}$ [By construction]

$\therefore \frac{\mathrm{DE}}{\mathrm{EA}}=\frac{\mathrm{OD}}{\mathrm{BO}}$[By Basic Proportionality theorem]

$\Rightarrow \frac{EA}{DE}=\frac{BO}{DO}$

$\Rightarrow \frac{EA}{DE}=\frac{BO}{DO}=\frac{AO}{CO}$

$\left[\because \frac{\mathrm{AO}}{\mathrm{CO}}=\frac{\mathrm{BO}}{\mathrm{DO}}\right]$

$\Rightarrow \frac{\mathrm{EA}}{\mathrm{DE}}=\frac{\mathrm{AO}}{\mathrm{CO}}$

Thus in $\triangle ADC, E$ and O are points dividing the sides AD and AC in the same ratio. Therefore by the converse of Basic Proportionality theorem, we have

$\mathrm{EO} \| \mathrm{DC}$

But EO \| AB[By construction]

$\therefore AB \| D$

$\therefore$ Quadrilateral ABCD is a trapezium

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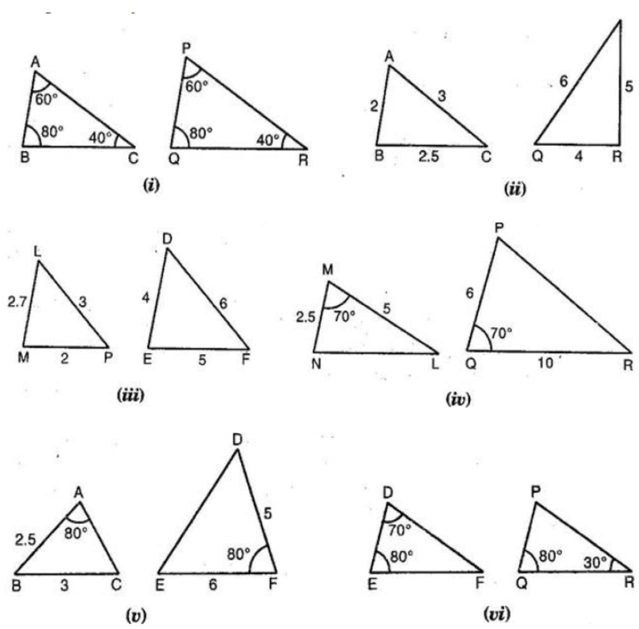
Chapter 6 Triangles NCERT Solutions Class 10 Maths: Simplify Concepts & Ace Exams

Ex 6.3 Question 1.

State which pairs of triangles in the given figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



Answer.

(i) In $\Delta s ABC$ and PQR, we observe that,

$\angle \mathrm{A} = \angle \mathrm{P} = 60^{\circ}, \angle \mathrm{B} = \angle \mathrm{Q} = 80^{\circ}$ and $\angle \mathrm{C} = \angle \mathrm{R} = 40^{\circ}$

$\therefore$ By AAA criterion of similarity, $\triangle \mathrm{ABC} \sim \triangle \mathrm{PQR}$

(ii) In $\triangle s ABC$ and PQR, we observe that,

$\frac{\mathrm{AB}}{\mathrm{QR}} = \frac{\mathrm{BC}}{\mathrm{RP}} = \frac{\mathrm{CA}}{\mathrm{PQ}} = \frac{1}{2}$

$\therefore$ By SSS criterion of similarity, $\triangle \mathrm{ABC} \sim \triangle \mathrm{PQR}$

(iii) In Δs LMP and DEF, we observe that the ratio of the sides of these triangles is not equal.

Therefore, these two triangles are not similar.

(iv) In $\triangle$s MNL and QPR, we observe that, $\angle \mathrm{M} = \angle \mathrm{Q} = 70^{\circ}$

But, $\frac{MN}{\mathrm{PQ}} \neq \frac{\mathrm{ML}}{\mathrm{QR}}$

$\therefore$ These two triangles are not similar as they do not satisfy SAS criterion of similarity.

(v) In $\triangle s ABC$ and FDE, we have, $\angle \mathrm{A} = \angle \mathrm{F} = 80^{\circ}$

But, $\frac{\mathrm{AB}}{\mathrm{DE}} \neq \frac{\mathrm{AC}}{\mathrm{DF}} [\because \mathrm{AC}$ is not given $]$

$\therefore$ These two triangles are not similar as they do not satisfy SAS criterion of similarity.

(vi) In $\triangle$s DEF and PQR, we have, $\angle \mathrm{D} = \angle \mathrm{P} = 70^{\circ}$

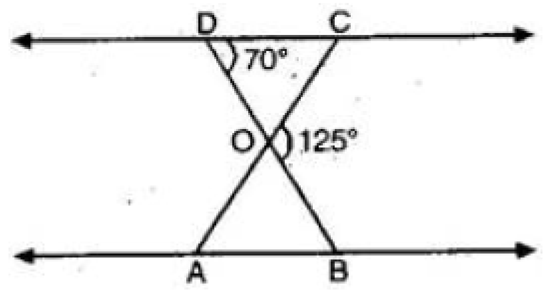
$[\because \angle \mathrm{P} = 180^{\circ} - 80^{\circ} - 30^{\circ} = 70^{\circ}]$

And $\angle \mathrm{E} = \angle \mathrm{Q} = 80^{\circ}$

$\therefore$ By AAA criterion of similarity, $\triangle \mathrm{DEF} \sim \triangle \mathrm{PQR}$

Ex 6.3 Question 2.

In figure, $\triangle ODC \sim \Delta OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Answer.

Since BD is a line and OC is a ray on it.

$\therefore \angle DOC + \angle BOC = 180^\circ$

$\Rightarrow \angle DOC + 125^\circ = 180^\circ$

$\Rightarrow \angle DOC = 55^\circ$

In ΔCDO, we have $\angle CDO + \angle DOC + \angle DCO = 180^\circ$

$\Rightarrow 70^\circ + 55^\circ + \angle DCO = 180^\circ$

$\Rightarrow \angle DCO = 55^\circ$

It is given that $\triangle ODC \sim \triangle OBA$

$\therefore \angle OBA = \angle ODC, \angle OAB = \angle OCD$

$\Rightarrow \angle OBA = 70^\circ, \angle OAB = 55^\circ$

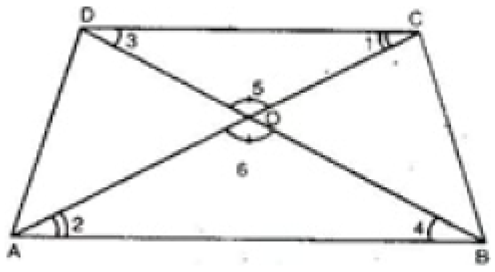
Hence $\angle DOC = 55^\circ, \angle DCO = 55^\circ$ and $\angle OAB = 55^\circ$

Ex 6.3 Question 3.

Diagonals AC and BD of a trapezium $ABCD$ with $AB \|$ | CD intersect each other at the point 0 . Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Answer.

Given: ABCD is a trapezium in which $\mathrm{AB} \| \mathrm{DC}$.



To Prove: $\frac{OA}{OC} = \frac{OB}{OD}$

Proof: In $\triangle s OAB$ and OCD, we have,

$\angle 5 = \angle 6$ [Vertically opposite angles]

$\angle 1 = \angle 2$ [Alternate angles]

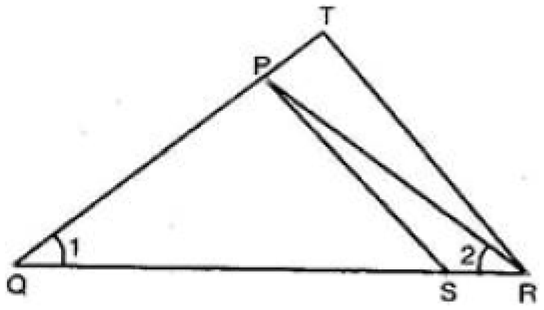
And $\angle 3 = \angle 4$ [Alternate angles]

$\therefore$ By AAA criterion of similarity, $\triangle OAB \sim \triangle ODC$

Hence, $\frac{OA}{OC} = \frac{OB}{OD}$

Ex 6.3 Question 4.

In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Answer.

We have, $\frac{QR}{QS} = \frac{QT}{PR}$

$\Rightarrow \frac{QT}{QR} = \frac{PR}{QS}$

Also, $\angle 1 = \angle 2$ [Given]

$\therefore PR = PQ$ (2) [$\because$ Sides opposite to equal $\angle$s are equal]

From eq.(1) and (2), we get

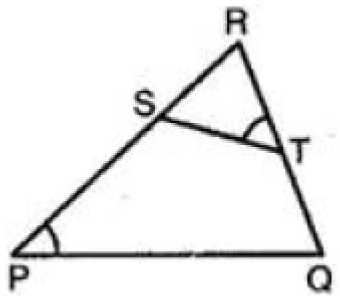
$\frac{QT}{QR} = \frac{PR}{QS} \Rightarrow \frac{PQ}{QT} = \frac{QS}{QR}$

In Δs PQS and TQR, we have,
$\frac{PQ}{QT}=\frac{QS}{QR}$ and $\angle PQS=\angle TQR=\angle Q$
$\therefore$ By SAS criterion of similarity, $\triangle PQS \sim \Delta TQR$
Ex 6.3 Question 5.

S and T are points on sides PR and QR of a $\triangle PQR$ such that $\angle \mathbf{P}=\angle$ RTS. Show that $\triangle \mathbf{RPQ} \sim \Delta \mathbf{RTS}$.

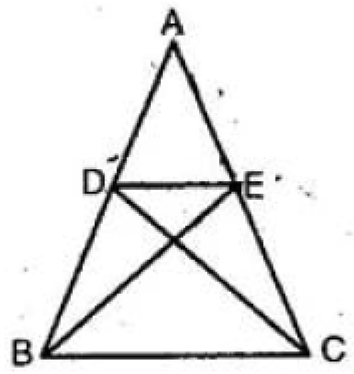
Answer.

In Δs RPQ and RTS, we have



$\angle RPQ=\angle RTS$ [Given]
$\angle PRQ=\angle TRS$ [Common]
$\therefore$ By AA-criterion of similarity,
$\triangle RPQ \sim \Delta RTS$
Ex 6.3 Question 6.

In the given figure, if $\triangle ABE \cong \triangle ACD$, show that $\triangle ADE \sim \triangle ABC$.

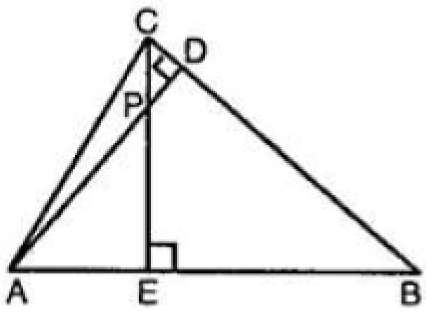


Answer.

It is given that $\triangle ABE \cong \triangle ACD$
$\therefore AB=AC$ and $AE=AD$
$\Rightarrow \frac{AB}{AD}=\frac{AC}{AE}$
$\Rightarrow \frac{AB}{AC}=\frac{AD}{AE}$..........(1)
$\therefore$ In $\triangle$s ADE and ABC, we have,
$\frac{AB}{AC}=\frac{AD}{AE}$[from eq.(1)]

And $\angle BAC=\angle DAE$[Common]
Thus, by SAS criterion of similarity, $\triangle ADE \sim \triangle ABC$
Ex 6.3 Question 7.

In figure, altitude AD and CE of a $\triangle ABC$ intersect each other at the point P. Show that:



(i) $\triangle$ AEP $\sim \Delta$ CDP

(ii) $\triangle ABD \sim \triangle CBE$
(iii) $\triangle AEP \sim \triangle ADB$
(iv) $\triangle$ PDC $\sim \triangle$ BEC

Answer.

(i) In Δs AEP and CDP, we have,
$\angle AEP=\angle CDP=90^{\circ}$[$\because$ CE $\perp$ AB, AD $\perp$ BC]
And $\angle APE=\angle CPD$[Vertically opposite]
$\therefore$ By AA-criterion of similarity, $\triangle$ AEP $\sim \triangle$ CDP
(ii) In $\triangle$sAABD and CBE, we have,
$\angle ADB=\angle CEB=90^{\circ}$

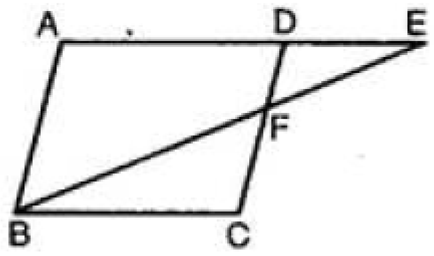
And $\angle ABD = \angle CBE$[Common]
$\therefore$ By AA-criterion of similarity, $\triangle ABD \sim \Delta CBE$
(iii) In Δs AEP and ADB, we have,
$\angle AEP = \angle ADB = 90^{\circ}[\because AD \perp BC, CE \perp AB]$
And $\angle PAE = \angle DAB$[Common]
$\therefore$ By AA-criterion of similarity, $\triangle AEP \sim \triangle ADB$
(iv) In Δs PDC and BEC, we have,
$\angle PDC = \angle BEC = 90^{\circ}[\because CE \perp AB, AD \perp BC]$
And $\angle PCD = \angle BEC$[Common]
$\therefore$ By AA-criterion of similarity, Δ PDC $\sim$ Δ BEC

Ex 6.3 Question 8.

E is a point on the side AD produced of a parallelogram $ABCD$ and BE intersects CD at F. Show that $\triangle ABE \sim \Delta CFB$.

Answer.

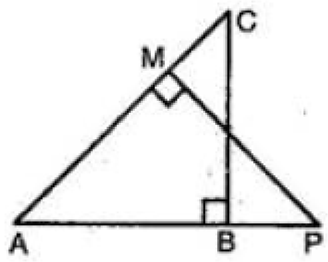
InΔs ABE and CFB, we have,



$\angle AEB = \angle CBF$[Alt. $\angle$s]
$\angle A = \angle c$ [opp. $\angle$s of a $\|$gm]
$\therefore$ By AA-criterion of similarity, we have
$\triangle ABE \sim \Delta CFB$

Ex 6.3 Question 9.

In the given figure, ABC and AMP are two right triangles, right angles at B and M respectively. Prove that:



(i) $\triangle ABC \sim \Delta AMP$
(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Answer.

(i) In $\Delta s ABC$ and AMP, we have,
$\angle ABC = \angle AMP = 90^{\circ}$ [Given]
$\angle BAC = \angle MAP$ [Common angles]
$\therefore$ By AA-criterion of similarity, we have
$\triangle ABC \sim \triangle AMP$

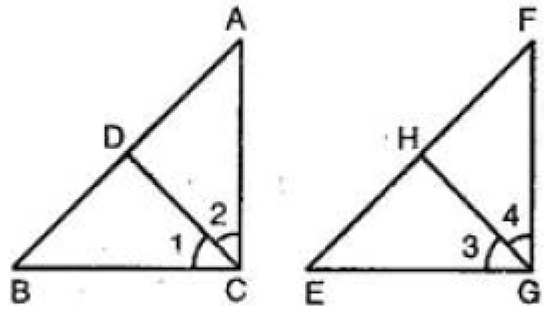
(ii) We have $\Delta ABC \sim \Delta AMP$ [As prove above]
$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$

Ex 6.3 Question 10.

CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE at $\triangle ABC$ and $\triangle EFG$ respectively. If $\triangle ABC \sim \triangle FEG$, show that:
(i) $\frac{CD}{GH} = \frac{AC}{FG}$
(ii) $\triangle \mathbf{DCB} \sim \triangle HE$
(iii) $\triangle \mathbf{DCA} \sim \Delta$ HGF

Answer.

We have, $\triangle ABC \sim \Delta FEG$



$\Rightarrow \angle A = \angle F \ldots\ldots\ldots (1)$

And $\angle C = \angle G$

$\Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle G$

$\Rightarrow \angle 1 = \angle 3$ and $\angle 2 = \angle 4 \ldots\ldots\ldots (2)$

[$\because$ CD and GH are bisectors of $\angle C$ and $\angle G$ respectively]

$\therefore$ In Δs DCA and HGF, we have

$\angle A = \angle F$[From eq.(1)]

[$\because$ CD and GH are bisectors of $\angle C$ and $\angle G$ respectively]

$\therefore$ In $\triangle$s DCA and HGF, we have

$\angle 2 = \angle 4$[[From eq.(2)]

$\therefore$ By AA-criterion of similarity, we have

$\triangle DCA \sim \triangle HGF$

Which proves the (iii) part

We have, $\triangle DCA \sim \Delta HGF$

$\Rightarrow \frac{AG}{FG} = \frac{CD}{GH}$

$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$

Which proves the (i) part

In Δs DCA and HGF, we have

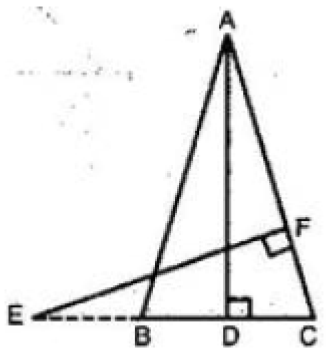
$\angle 1 = \angle 3$ [From eq.(2)]

$\angle B = \angle E [\because \Delta DCB \sim \Delta HE]$

Which proves the (ii) part

Ex 6.3 Question 11.

In the given figure, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.



Answer.

Here $\triangle ABC$ is isosceles with $AB = AC$

$\therefore \angle B = \angle C$

In $\Delta s ABD$ and ECF, we have

$\angle ABD = \angle ECF [\because \angle B = \angle C]$

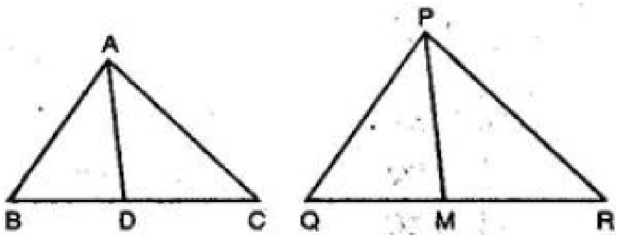
$\angle ABD = \angle ECF = 90^\circ [\because AD \perp BC$ and $EF \perp AC]$

$\therefore$ By AA-criterion of similarity, we have

$\triangle ABD \sim \triangle ECF$

Ex 6.3 Question 12.

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of a $\triangle PQR$ (see figure). Show that $\triangle ABC \sim \triangle PQR$.



Answer.

Given: AD is the median of $\triangle ABC$ and PM is the median of $\triangle PQR$ such that $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

To prove: $\triangle ABC \sim \Delta PQR$

Proof: $BD = \frac{1}{2}BC$ [Given]

And $QM = \frac{1}{2}QR$ [Given]

Also $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ [Given]

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$\therefore \Delta ABD \sim \Delta PQM$[By SSS-criterion of similarity]

$\Rightarrow \angle B = \angle Q$[Similar triangles have corresponding angles equal]

And $\frac{AB}{PQ} = \frac{BC}{QR}$[Given]

And $\frac{AB}{PQ} = \frac{BC}{QR}$ [Given]

∴ By SAS-criterion of similarity, we have

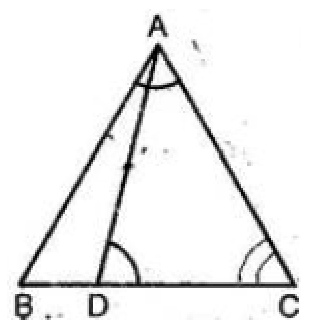
$\triangle ABC \sim \Delta PQR$

Ex 6.3 Question 13.

D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = \mathbf{CB}.CD$.

Answer.

In triangles ABC and DAC,



$\angle ADC = \angle BAC$ [Given]

and $\angle c = \angle C$[Common]

∴ By AA-similarity criterion,

$\triangle ABC \sim \triangle DAC$

$\Rightarrow \frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$

$\Rightarrow \frac{CB}{CA} = \frac{CA}{CD}$

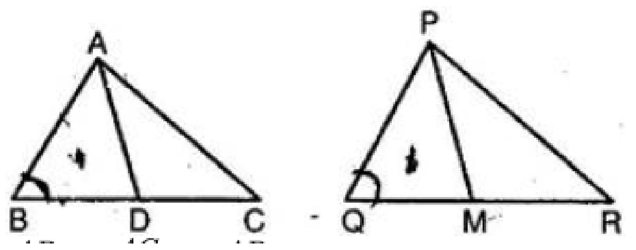
$\Rightarrow CA^2 = CB.CD$

Ex 6.3 Question 14.

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$.

Answer.

Given: AD is the median of $\triangle ABC$ and PM is the median of $\triangle PQR$ such that

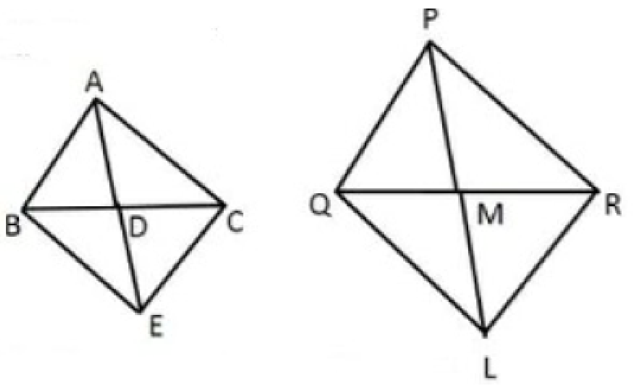


$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \ldots\ldots\ldots(1)$

To prove: $\Delta ABC \sim \Delta PQR$

Proof:

Let us extend AD to point D such that $AD = DE$ and PM up to point L such that $PM = ML$



Join B to E. C to E,and Q to L, and R to L

We know that medians is the bisector of opposite side

Hence

$BD = DC$

Also, $\mathrm{AD}=\mathrm{DE}$ (by construction)

Hence in quadrilateral ABEC, diagonals AE and BC bisect each other at point D. Therefore, quadrilateral ABEC is a parallelogram.

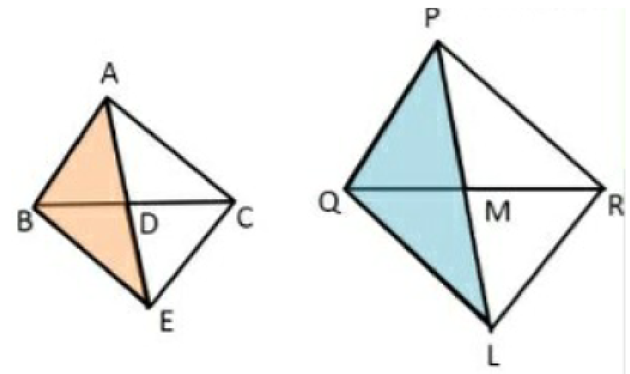
$$\mathrm{AC}=\mathrm{BE}$$
$$\mathrm{AB}=\mathrm{EC} \text{ (opposite sides of } \backslash| \text{ |gm are equal)}$$

Similarly, we can prove that PQLR is a parallelogram

$$PR=QL$$
$$PQ=LR \text{ opposite sides of } \| \text{ gm are equal)}$$

Given that

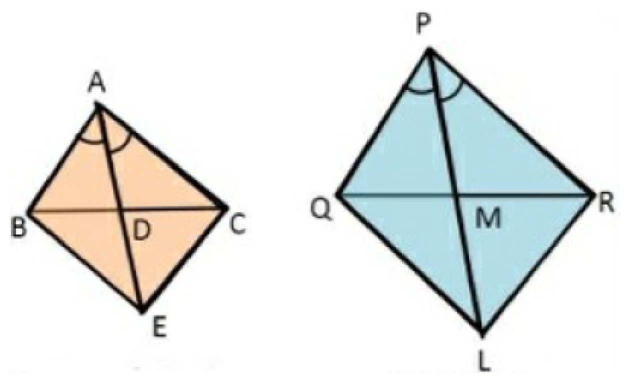
$$\frac{AB}{PQ}=\frac{AC}{PR}=\frac{AD}{PM}$$
$$\frac{AB}{PQ}=\frac{BE}{QL}=\frac{AD}{PM}[\text{ from (2) (3)]}$$
$$\frac{AB}{PQ}=\frac{BE}{QL}=\frac{2AD}{2PM}$$
$$\frac{AB}{PQ}=\frac{BE}{QL}=\frac{AE}{PL}[\text{ as } AD=DE, AE=AD+DE=2AD$$
$$PM=ML \cdot PL=PM+ML=2PM$$
$$\triangle ABE \sim \triangle PQL \quad \text{(By SSS Similiarity Criteria)}$$



We know that corresponding angles of similar triangles are equal.

$\angle BAE=\angle QPL(4)$

Similarly, we can prove that $\triangle AEC \sim \triangle PLR$.

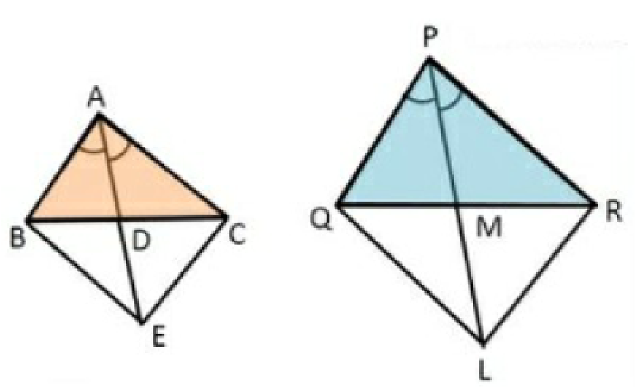


We know that corresponding angles of similar triangles are equal.

$\angle CAE=\angle RPL$ (5)

Adding (4) and (5),

$$\angle BAE+\angle CAE=\angle QPL+\angle RPL$$
$$\angle CAB=\angle RPQ$$
$$\text{In } \triangle ABC \text{ and } \triangle PQR$$



$$\frac{AB}{PQ}=\frac{AC}{PR}$$
$$\angle CAB=\angle RPQ$$
$$\triangle ABC \sim \triangle PQR$$

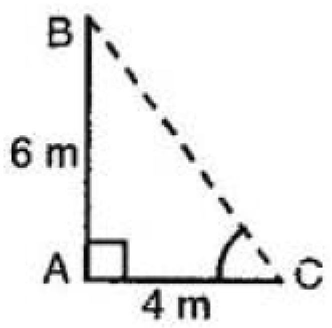
Hence proved

Ex 6.3 Question 15.

A vertical pole of length $6 \mathrm{~m}$ casts a shadow $4 \mathrm{~m}$ long on the ground and at the same time a tower casts a shadow $28 \mathrm{~m}$ long. Find the height of the tower.

Answer.

Let AB the vertical pole and AC be its shadow. Also, let DE be the vertical tower and DF be its shadow. Joined BC and EF.

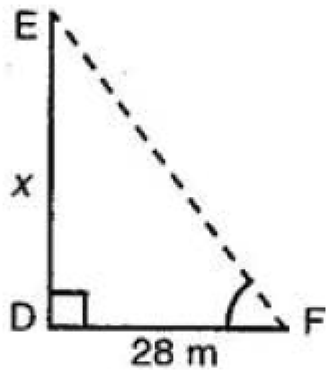


Let $\mathrm{DE}=x$ meters

Here, $\mathrm{AB}=6 \mathrm{~m}, \mathrm{AC}=4 \mathrm{~m}$ and $\mathrm{DF}=28 \mathrm{~m}$

In the triangles ABC and DEF,
$\angle \mathrm{A}=\angle \mathrm{D}=90^{\circ}$

And $\angle \mathrm{C}=\angle \mathrm{F}$ [Each is the angular elevation of the sun]



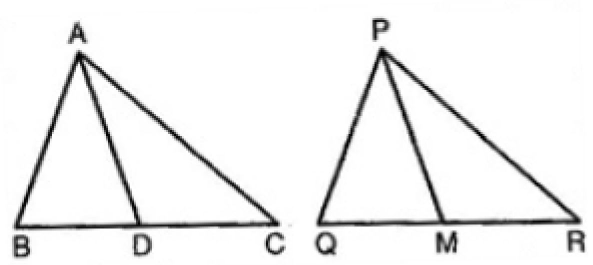
$\therefore$ By AA-similarity criterion,
$\Delta \mathrm{ABC} \sim \Delta \mathrm{DEF}$
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{DE}}=\frac{\mathrm{AC}}{\mathrm{DF}}$
$\Rightarrow \frac{6}{x}=\frac{4}{28}$
$\Rightarrow \frac{6}{x}=\frac{1}{7}$
$\Rightarrow x=42 \mathrm{~m}$

Hence, the height of the tower is 42 meters.
Ex 6.3 Question 16.

If AD and PM are medians of triangles ABC and PQR respectively, where $\triangle \mathrm{ABC} \sim \Delta$ PQR, prove that $\frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\mathrm{AD}}{\mathrm{PM}}$

Answer.

Given: AD and PM are the medians of triangles
ABC and PQR respectively, where



$\triangle \mathrm{ABC} \sim \Delta \mathrm{PQR}$
To prove: $\frac{A B}{P Q}=\frac{A D}{P M}$
Proof: In triangles ABD and PQM,
$\angle \mathrm{B}=\angle \mathrm{Q}$ [Given]
And $\frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\frac{1}{2} \mathrm{BC}}{\frac{1}{2} \mathrm{QR}}$[$\because$ AD and PM are the medians of BC and QR respectively]
$\Rightarrow \frac{A B}{P Q}=\frac{B D}{Q M}$
$\therefore$ By SAS-criterion of similarity,
$\triangle \mathrm{ABD} \sim \triangle \mathrm{PQM}$

$\Rightarrow \frac{A B}{P Q}=\frac{B D}{Q M}=\frac{A D}{P M}$
$\Rightarrow \frac{A B}{P Q}=\frac{A D}{P M}$